Macro-financial feedbacks through time*

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Abstract

Changing (co-)variances of macroeconomic and financial series provide strong identification power in disentangling real-financial interactions. "Identification through heteroskedasticity" assumes changing (co-)variances stem only from changing structural shock-volatility. This paper generalizes the approach to encompass time-varying parameters. Imposing as constant either coefficients or shock volatilities does not reproduce real-financial (co-)variances for the US. The set of structural models that match the data contains both models with negative feedbacks and boom-bust theories. Alternative identification approaches unduly exclude plausible theories. The elasticity of financial to real variables increased around the 2000's, while that of real to financial variables fell.

Keywords: Heteroskedasticity, Non-recursive SVAR, Macro-financial Feedbacks

JEL Codes: C32, C51, E44

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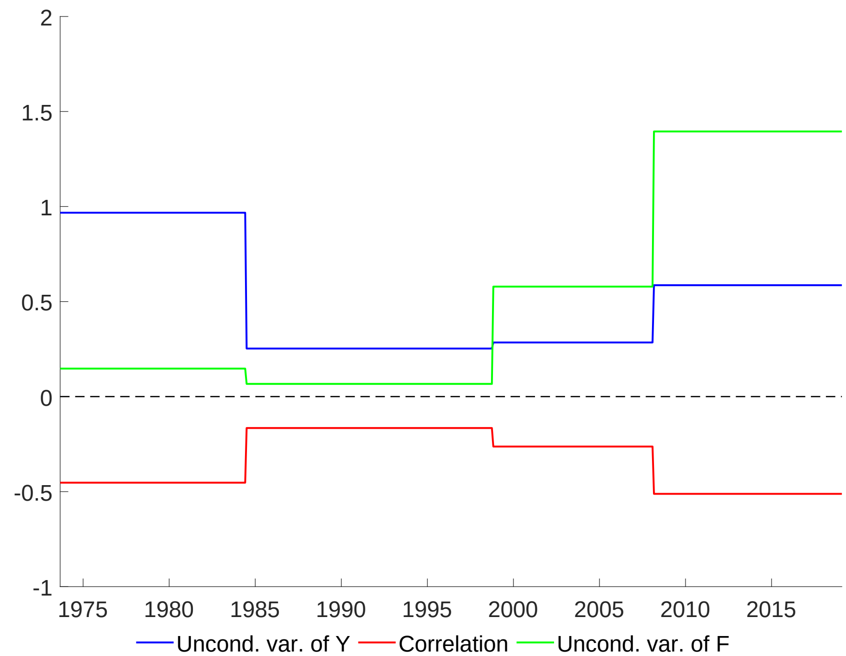
1 Introduction

Following the Global Financial Crisis (GFC henceforth) macro-financial linkages have taken center stage. The origins of the crisis were rooted in financial markets, with dire macroeconomic consequences, which then fed back to the financial sector, and so on. The macroeconomic impact was both severe and clearly distinct from co-movements between the two sectors in less turbulent times, marking a significant break from the macro-financial fluctuations observed during the Great Moderation (GM henceforth).

Figure 1 plots the unconditional volatilities and correlation of a real and a financial aggregate over the period 1973-2019, for four distinct subsamples, described below. We focus on data from the United States for industrial production and a measure of financial stability, namely the Gilchrist-Zakrajšek (GZ henceforth) spread. The chart testifies to the fact that both real and financial volatility exhibit stark fluctuations over time. The first period, from the ‘70s into the early ‘80s was quite turbulent, especially at the macroeconomic level, but to some extent also in financial markets. The second period witnessed a reduction in real volatility starting from the mid ‘80s. During these years financial volatility, too, was markedly low. The stability of macroeconomic fluctuations continued to the brink of the GFC. Financial volatility, however, rose substantially much earlier: in the late ‘90s/early ‘00s financial volatility ticked up quite significantly, apparently with little, if any, change in real volatility. The fourth period, starting with the GFC was defined by a substantial increase in both financial and macroeconomic volatility. During these episodes, the covariance between macroeconomic and financial variables has also varied. While it is negative throughout, after the GM and before the GFC the correlation was only mildly negative.

Understanding the changing nature of macro-financial dynamics is high on the macroeconomic policy agenda, but it presents both the economist and the econometrician with several challenges. First of all, even in a world of constant volatilities, one needs credible identification assumptions to unbundle the covariance between real and financial variables. The search for such identification assumptions permeates virtually all of structural macroeconometrics. Specifically, if the econometrician were to analyze the two variables using a structural VAR (or a variant of it), she would need to impose identifying assumptions to decompose the reduced form error variance-covariance matrix into contemporaneous interactions and structural volatilities. Second, in a world of changing volatilities and covariances, there is concern that any identifying assumptions used may not be time-invariant. That is, recognising that both parameters and volatilities can change through time implies needing to solve the identification problem in every single regime.

Figure 1: Unconditional variances and correlation of Δ industrial production (Y) and GZ-spread (F), 4 subperiods.



The first papers in the literature dealing with changing macro-financial dynamics have essentially assumed the problem away, by imposing recursivity on the system (a Choleski structure, where the financial block can respond contemporaneously to the macro block, but not vice versa). Examples include Davig and Hakkio (2010), Hubrich and Tetlow (2015), Prieto et al. (2016). While surely a convenient approach, there is a general discontent in using such a recursive structure, both in general (e.g. Baumeister and Hamilton, 2019) and specifically for the question of macro-financial feedbacks, where effectively it assumes one type of feedback to be absent. Some more recent work aims at relaxing that stringency and admits some time-varying identification assumptions. Where exactly one allows such time variation is, however, largely arbitrary. For instance, Brunnermeier, Palia, Sastry and Sims (2021) assume it is present only in the shock volatilities, while the contemporaneous and autoregressive coefficients are time-invariant. Angelini, Bacchiocchi, Caggiano and Fanelli (2019) allow some elements of the system to change over time, but assume others fixed. Assuming any parameter constant a priori is potentially troubling because of the immediate impact it may have on structural conclusions, but without doing so one cannot identify structural shocks.

This is where our paper contributes. We extend the method of identification through heteroskedasticity to a setting in which not just shock volatilities, but also parameters can change through time. Mathematically this creates a set of structural models rather than a single point identified model. Economic theory provides very little guidance as to restricting this set, so contrary to the literature we refrain from imposing further a priori assumptions on it. Instead, we first show that within this set, a whole class of structural models fail to match the (co-)variance patterns in Figure 1. More precisely, any model that assumes that either contemporaneous coefficients, autoregressive coefficients or structural shock volatilities are constant through time does not deliver. Second, among the models where time variation is allowed in (parts of) these three blocks, a subset of them is rejected on purely empirical grounds: we compare each model's log-likelihood to that of a fully flexible model, where all coefficients and shock volatilities are allowed to change in each subperiod. A simple likelihood ratio test allows us to exclude a subset of these models, which fail comparison with the nesting "fully flexible" model.

The rest of the models, passing the likelihood ratio test, form a set with quite different assumptions on time variation, all of which succeed in capturing the changing macro-financial dynamics through time. That is, they differ on exactly which elements are allowed to vary from one period to the next. Even though this set of models may look quite disperse to the econometrician's eye, we show that the structural economic conclusions they give rise to are informative in their own right. As a result, for the economist aiming to answer structural questions, the multiplicity of nearly observationally equivalent models is not an issue, and perhaps even a virtue.

The structural conclusions that we draw are threefold.

Time-varying volatility Macro shock volatility decreased considerably throughout the Great Moderation and picked up again after the GFC, albeit not to the levels of the beginning of the sample. Financial shock volatility stayed low until the end of the 90s, then doubled and tripled in the third (2000-2008) and fourth (2009-2019) period, respectively. These results are in line with the literature so far. Yet our analysis shows they hold in a much broader class of models, with potentially very different identifying assumptions.

Feedbacks Our analysis reveals that the data in Figure 1 does not unequivocally pin down the sign of macro-financial feedbacks. We show this result is in fact consistent with theoretical models of financial frictions used to understand the GFC. Models with traditional feedbacks in which financial shocks are contractionary and in which positive real shocks reduce financial spreads are within the set of admissible models. But so are models in which one of the feedbacks is positive. For instance, one strand of models that features positive feedbacks are models that feature boom-bust cycles. Such models imply that increased spreads (e.g. through riskier borrowers) go hand in hand with increased investment or economic activity, at least in the short run. One implication of our results is that ruling out such models a priori (e.g. through Choleski or sign restrictions) could well imply policy implications which are not warranted by the data.

Time variation in feedbacks Our results further suggests that there is scope for models in which feedbacks changed over time, most markedly from period two to three. There is particular promise for models in which the financial impact of real shocks became stronger in the second half of our sample, while the real impact of financial shocks became more subdued. Our analysis comes with a number of caveats. Perhaps most importantly, we draw these conclusions based on models with a small number of variables, with time variation allowed in pre-defined periods. This enables us to assess the importance of different assumptions on the sources of time variation. The issues we highlight will most likely also be at play in larger models, or models with endogenously identified breaks. We provide a first check for this claim in the three-variable extension of Section 7, with results largely similar to the baseline specification. Overfitting might be yet another concern of particularly flexible models. We argue that limiting the time variation to four subperiods is a first way to limit this risk, and we also show how our relatively more flexible models are preferred (by a likelihood ratio test) to models with constant autoregressive coefficients (i.e. more restrictive).

The rest of the paper is organised as follows. Section 2 describes the identification method and the set of estimated models and Section 3 lists the data used. Section 4 documents that among the range of models, only a smaller set manages to successfully replicate the volatility and covariance dynamics of both macro and financial variables. It then documents the wide range of structural results across the set of selected models. Section 5 discusses implications for DSGE models and Section 7 describes the relation to traditional identification approaches and robustness analysis. Finally, Section 8 concludes.

2 Methodology

2.1 Standard identification through heteroskedasticity

Consider the following VAR model with constant parameters and time-varying volatilities:

$$A_0 y_t = A_1 y_{t-1} + ... + A_p y_{t-p} + C + \Lambda_t^{1/2} \varepsilon_t \qquad \varepsilon_t \sim N(0, I_n) \tag{1}$$

where y_t is a $n \times 1$ vector of observed variables, A_0 is a $n \times n$ matrix of contemporaneous relationships, $A_1, ..., A_p$ are matrices of lagged coefficients, C is a vector of constants, ε_t is a vector of independent shocks with identity variance covariance matrix, and $\Lambda_t^{1/2}$ is a diagonal matrix with the structural shocks' standard deviations on the diagonal.

In the traditional VAR model with constant volatilities, that is $\Lambda_t = \Lambda$, a well-known identification issue arises: there are n^2 unknown structural parameters in A_0 and Λ, but only $\frac{n(n+1)}{2}$ empirical moments in the variance covariance matrix of reduced form shocks, Σ. Additional identifying assumptions are needed to solve

$$\Sigma = A_0^{-1} \Lambda A_0^{-1\prime} \tag{2}$$

which may take the form of zero, long-run or sign restrictions, among the most common.

In the presence of heteroskedasticity, however, identification is possible without additional traditional identification assumptions, provided that structural volatilities do not change proportionally between regimes and the coefficients remain constant (see Rigobon, 2003). In this approach, the identification problem is solved as a result of two features. On the one hand, the presence of heteroskedasticity (multiple Σ) increases the amount of reduced form information available (on the left hand side of (2)). In the simplest, two-period example below, heteroskedasticity implies the reduced form variance-covariance matrix Σ takes different values in each period: Σ_1 and Σ_2.

$$\begin{aligned} \Sigma_1 &= A_0^{-1} \Lambda_1 A_0^{-1\prime} \\ \Sigma_2 &= A_0^{-1} \Lambda_2 A_0^{-1\prime} \end{aligned} \tag{3}$$

On the other hand, the assumption that any time variation in structural elements must come from changes in structural shock volatility (Λ_1, Λ_2), and not from changes in A_0, implies that the number of parameters on the right hand side of (3) does not increase proportionately.

Under these assumptions identification through heteroskedasticity then pins down a solution (e.g. Lanne et al., 2010; Lewis, 2021):

$$\Sigma_2^{-1} \Sigma_1 = A_0' \Lambda_2^{-1} \Lambda_1 A_0^{-1\prime} \tag{4}$$

takes the form of an eigenvalue decomposition, where the columns of A_0 are the eigenvectors and the elements of $\Lambda_2^{-1} \Lambda_1$ the eigenvalues. The eigenvalues need to be unique for the elements of A_0 to be identified (up to sign and row permutation), which means that the relative volatilities are not constant across regimes.[1] When the number of subperiods or regimes is higher than two, it can be shown that the model in (1) is over-identified. The second row of Table 1 provides a numerical example of this overidentification, for time-varying Λ and constant A_0.

2.2 An extension: identification through time variation

Identification through heteroskedasticity avoids imposing economic restrictions and is particularly popular in a setting of macro-financial feedbacks, where convincing a priori restrictions are hard to come by. One evident shortcoming is the necessity for A_0 to be constant, which is in itself an assumption on the time-invariant nature of transmission mechanisms.

This assumption is at odds with many DSGE models of macro-financial feedbacks that describe the GFC, such as models with occasionally binding constraints (e.g. Guerrieri and Iacoviello, 2017; Lindé, Smets and Wouters, 2016). The assumption also has stark policy implications, as several works on the Great Moderation or in the uncertainty literature indicate.[2] Against this background, it seems particularly unwarranted to impose constant feedbacks a priori.

Allowing feedbacks to change across subperiods s suggests estimating:

$$A_{0,s} y_t = A_{1,s} y_{t-1} + ... + A_{p,s} y_{t-p} + \Lambda_s^{1/2} \varepsilon_t \qquad \varepsilon_t \sim N(0, I) \tag{5}$$

rather than (1), which permits time variation in all three blocks of coefficients: $A_{0,s}$, $A_{j,s}$ ($j = 1, ..., p$) and Λ_s. The matrix $A_{0,s}$ of contemporaneous coefficients is estimated in terms of changes with respect to the previous period: in the first period, $A_{0,1}$, in the second period, $A_{0,2} = A_{0,1} + Q$, where Q is a matrix of same dimensions as $A_{0,1}$; the matrix $A_{j,s}$ of autoregressive coefficients is estimated freely in each period, i.e. $A_{j,1}$, $A_{j,2}$, ... $A_{j,s}$ are uncorrelated across periods; finally, the matrix Λ_s of shocks volatilities is imposed to be non-zero in the first period (meaning that shocks volatilities are always estimated in the first period, and then might vary or stay constant in the subsequent periods, depending on whether volatilities are re-estimated or a zero restriction is imposed).

[1] A restriction is necessary to exclude scaling of Λ or A_0. This can be obtained by setting $\Lambda_1 = 1$ (as in, e.g., Herwartz and Lütkepohl, 2014), or by imposing the average of Λ_t across periods equal to one (as in Brunnermeier et al., 2021), or by normalising the diagonal of A_0 to be equal to one in every subperiod, as we do below.

[2] Cogley and Sargent (2001, 2005); Lubik and Schorfheide (2004); Sims and Zha (2006); Baele, Bekaert, Cho, Inghelbrecht and Moreno (2015); Caggiano, Castelnuovo and Groshenny (2014); Alessandri and Mumtaz (2019); Carriero, Clark and Marcellino (2018a,b).

Table 1: Number of moments and parameters for different model specifications.

Model	# moments	# parameters	Numerical example
Constant model	$\frac{n(n+1)}{2}$	$n(n-1)+n$	3 moments < 4 params
Time-varying Λ	$s\frac{n(n+1)}{2}$	$sn(n-1)+n$	12 moments > 10 params
Time-varying A_0 and Λ	$s\frac{n(n+1)}{2}$	$s[n(n-1)+n]$	12 moments < 16 params

Note: n is the number of variables, s is the number of subperiods or regimes. The numerical example refers to the case where $n = 2$ and $s = 4$.

Time variation in coefficients poses a problem for identification through heteroskedasticity, however. Once A_0 is allowed to change across periods, (3) generally admits multiple solutions (beyond normalisation and row permutations). Consider the two-period case once more, where now we allow for time-varying $A_{0,s}$:

$$\begin{aligned} \Sigma_1 &= A_{0,1}^{-1}\Lambda_1 A_{0,1}^{-1\prime} \\ \Sigma_2 &= A_{0,2}^{-1}\Lambda_2 A_{0,2}^{-1\prime} \end{aligned} \tag{6}$$

which implies

$$\Sigma_2^{-1}\Sigma_1 = A_{0,2}'\Lambda_2^{-1}A_{0,2}A_{0,1}^{-1}\Lambda_1 A_{0,1}^{-1\prime}. \tag{7}$$

Clearly the information in Σ_1 and Σ_2 no longer suffices to pin down the structural parameters and volatilities, i.e. the matrices $A_{0,1}$, $A_{0,2}$, Λ_1, Λ_2.[3] The last row of Table1 gives an example with two variables and four subperiods.

Some papers have addressed this by picking a single solution to (7), e.g. Angelini et al. (2019) and Bacchiocchi and Fanelli (2015). They argue in favour of specific zero restrictions and time-constant parameters in $A_{0,s}$, and impose those a priori. We find it very hard to come by convincing economic restrictions that plausibly narrow down the set of solutions. Economic theory provides very little guidance in the matter. Both zero restrictions and parameter-constancy are scarce in recent DSGE models of macro-financial feedbacks. In light of that, rather than arguing for a single solution, we study the entire set of solutions to (7). More precisely, we estimate in structural form all possible versions of (5) subject to the constraint that the number of elements that are estimated in $A_{0,s}$ and Λ_s ($\forall s$) is equal to the number of moments that can be obtained from Σ_s ($\forall s$), so that each model is exactly identified.

[3]Note that time variation in lagged coefficients $A_{j,s}$ (for $j = 1, ..., p$) does not affect the identification problem directly, but is relevant through the impact it has on the estimation of Σ_s.

2.3 Suite of models and estimation

We estimate by maximum likelihood a suite of two-variable structural vector autoregression (SVAR) models such as the one in (5), with one real sector variable and one financial sector one. We assume a standard Gaussian log-likelihood for each regime:

$$log(L_s) = -\frac{NT}{2} log\left(2\pi det(\Sigma_s)\right) - \frac{1}{2}\sum_{j=1}^{T} u_j'\Sigma_s^{-1}u_j \tag{8}$$

where s is the subscript indicating the regime or subperiod, N is the number of variables, T is the number of observations in each subperiod, $\Sigma_s = A_{0,s}^{-1}\Lambda_s A_{0,s}^{-1'}$ is the variance covariance matrix of reduced form shocks and $u_j = y_j - \beta_s x_j$, with $\beta_s = A_{0,s}^{-1}A_{j,s}$. Each maximisation problem is a separate one in each period, with the likelihoods only linked by the fact that the parameters in $A_{0,s}$ are connected across periods via the following: $A_{0,2} = A_{0,1} + Q_2$, $A_{0,3} = A_{0,1} + Q_2 + Q_3$, and so on, where Q_2, Q_3 are matrices of the same dimension as A_0. We use a standard unconstrained maximisation algorithm. The models all have the same lag structure ($p = 4$), but they differ as to where and when time variation is allowed. For example, one model might have different contemporaneous coefficients in all the four subperiods of the sample, but constant variance for one of the shocks; another model might have time-varying variances and constant transmission mechanism of the real to the financial sector. We do not select where time variation lies a priori, instead we estimate all possible combinations.

2.3.1 Number of estimated models

We first calculate the total number of parameters and moments, in order to determine how many restrictions are necessary for the model to be identified.

In total, we have $n(n-1)s + ns$ parameters. In A_0, at most we can estimate $n(n-1)s$ parameters, where n is the number of variables and s the number of subperiods.[4] In Λ, we always estimate the first period volatilities (n), and at most we can have ns estimated parameters.

We have $s\frac{n(n+1)}{2}$ moments, which corresponds to the maximum number of parameters to estimate. In the numerical example in the last row of Table 1, this implies 16 parameters and 12 moments. We can then estimate 12 parameters, to be assigned randomly to 16 locations either in A_0 or in Λ. This corresponds to $(16 - 12) = 4$ zero restrictions across

[4] Recall that the diagonal of A_0 is set equal to one in all subperiods.

A_0 and Λ and all subperiods.

We establish the number of possible model by means of a simple combination, $\frac{n!}{k!(n-k)!}$, where n is the number of possible locations, 16, and k the number of restrictions needed for each model to be identified, 4. Given our numbers, we have 1001 possibilities, corresponding to as many models to estimate.

3 Data and subperiods

In our main specification the real sector variable is the monthly growth rate of U.S. real industrial production,[5] while for the financial sector we use the corporate bond spread of Gilchrist and Zakrajšek (GZ spread).[6] A word of caution is necessary regarding the financial variable: there are many aspects of financial markets that might affect the real economy differently; often, papers use composite indicators such as the Financial Condition Index by the Chicago Fed, or different spreads like the term or the safety spread. Each indicator can be more or less connected to financial market developments, and more or less correlated to the business cycle. As we show in Figures 13 and 13 in the Appendix, however, similar dynamics seem to hold for several other financial indicators.

The data goes from January 1973 to the end of 2018. Based on well known events and on the volatilities dynamics shown in Figure 1, we split the sample into four subperiods: the "Great Inflation", from the start of the sample till mid-1984, followed by the long "Great Moderation", interrupted at the start of the century by a third period characterised by high financial volatility (caused, among others, by the burst of the "dot-com bubble"). The final subperiod includes the "Great Recession" of 2008 and the recovery experienced by the US after the crisis. Brunnermeier et al. (2021) also use fixed period dates and select seven distinct regimes, based on changes in the volatilities of ten data series; Angelini et al. (2019) only have three regimes, with the Great Moderation assumed to last until the eve of the GFC. We follow closely the latter regime division, further adding the "dot-com bubble" period due to the striking break in financial volatility (see Figure 1) coupled with little change in the real one. Note that the exact cutoff date for each subperiod does not affect estimation consistency even if it does not correspond exactly to the date when volatilities changed, as long as the average volatilities in each period are different and do not change proportionally, as explained in Sims (2020).

[5]From the FRED database.

[6]See Gilchrist and Zakrajšek (2012). Available at https://www.federalreserve.gov/econresdata/notes/feds-notes/2016/files/ebp_csv.csv

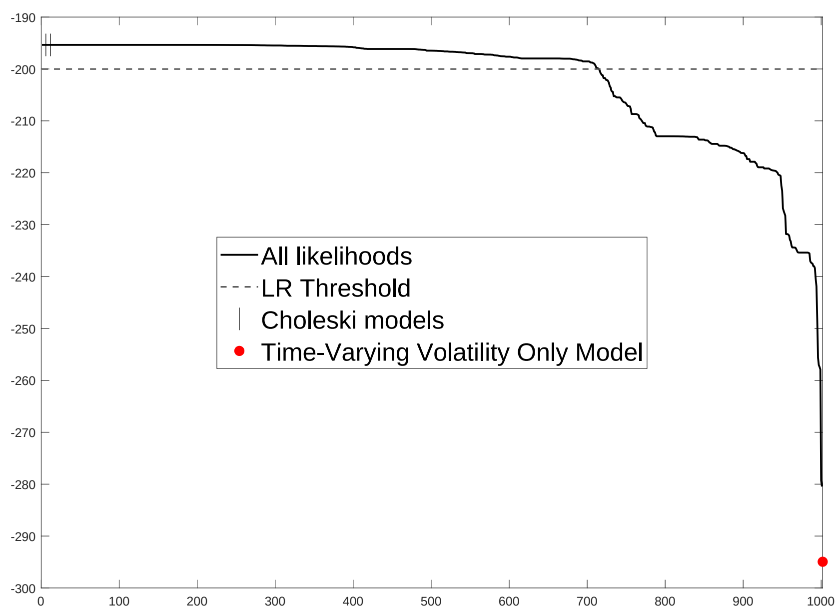
4 Model Selection and Results

We proceed in three steps. First, we show that one can reject a subset of estimated models on purely empirical (reduced form) grounds. We then disregard those models and zoom in on the set of models that does succeed in matching the data. Second, to make these models comparable for structural inference, we address the issue of permutations. Third, we present the structural implications of the set of successful, comparable models.

4.1 Model selection

The log-likelihood of each model is one measure which can be used for selection, provided that the compared models are nested.[7]

Figure 2: Estimated log-likelihoods for 1001 estimated models.

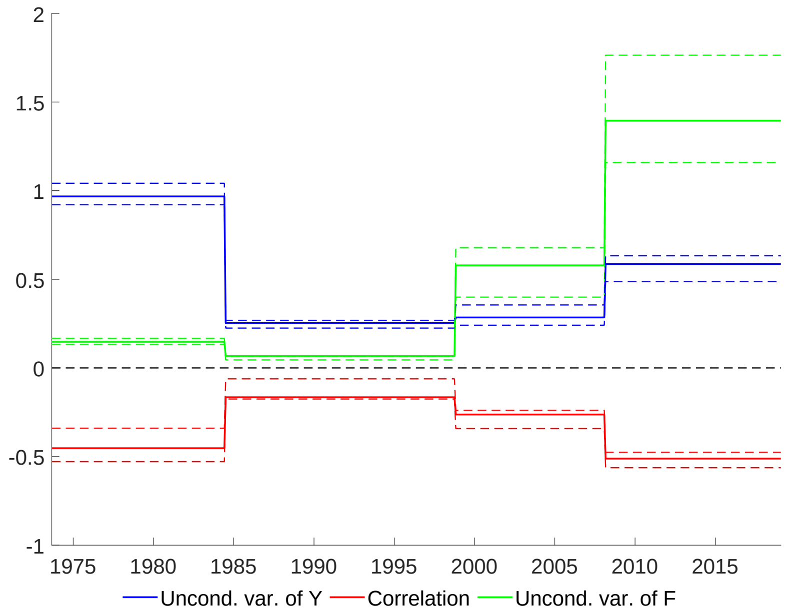


Note: The dashed line is the threshold of the likelihood ratio test and the black dashes indicate the positions of the two Choleski models.

Figure 2 shows the estimated likelihoods for all models, sorted from highest to lowest, or best to worst. There is a clear deterioration in the likelihood beyond a threshold (see the dashed line). Given that one important feature of any empirical model is the ability to fit the data well, we only focus on the models with the best likelihoods. In order to select them,

[7]With nested models, it is possible to "penalise" the most flexible one for the additional degrees of freedom.

Figure 3: Unconditional variances and correlation of Δ industrial production (Y) and GZ-spread (F).

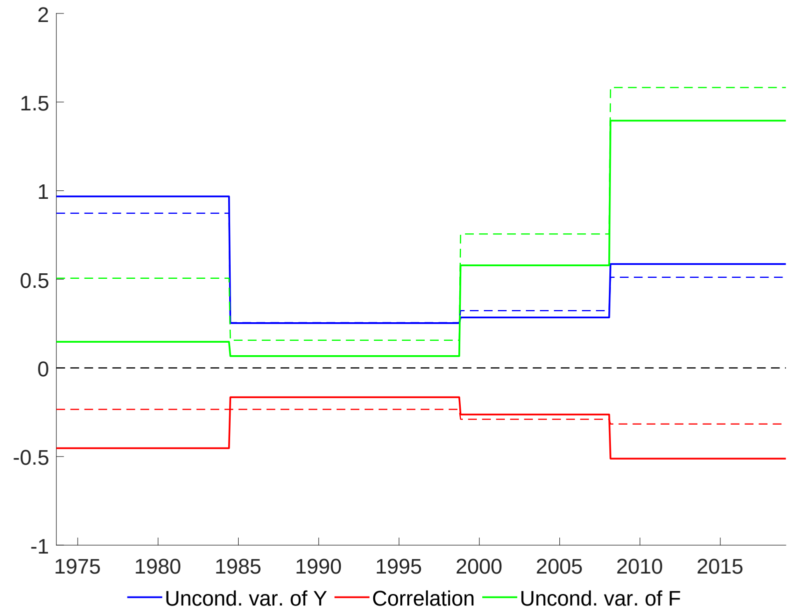


Note: Range of models (dashed lines), data (solid lines).

we perform a likelihood ratio test between each model and a model where all coefficients and shocks volatilities are assumed to be time-varying. This "general" model is itself unidentified, but nests all the identified models whose likelihoods are plotted in Figure 2. The dashed horizontal line represents the threshold between the models passing and those failing the test at the 5% significance level. The black vertical dashes highlight the two models with a Choleski identification (where the matrix of contemporaneous coefficients, A_0, is forced to remain triangular across all the four subperiods). The other models above the dashed lines in Figure 2 are all models combining time variation in A_0 and in Λ for some or all of the subperiods, and a time-varying matrix of lagged coefficients A_j. All models above the horizontal line in Figure 2 are able to replicate the variances and correlation patterns of Figure 1. Figure 3 shows the unconditional variance and correlation bounds of the models that are not rejected by the likelihood ratio test (dashed lines), and the corresponding values from the data (solid lines). The models follow very closely all three moments in all four subperiods, with a larger dispersion across models only in the last two periods for the variance of the financial variable.

All models in Figure 2 feature time variation in A_0, Λ and A_j, while keeping a constant

Figure 4: Unconditional variances and correlation of Δ industrial production (Y) and GZ-spread (F).



Note: "Time-Varying Volatility only" model (dashed lines), data (solid lines).

number of estimated parameters. Models where either A_0, Λ or A_j are assumed constant do not succeed in replicating the (co-)variance pattern of Figure 1. Any such model is also rejected by a likelihood ratio test relative to the unidentified model where everything varies. Table 2 provides likelihood comparisons for a subset of these models: namely, it compares models with constant A_j matrices and models where A_j vary in each period. The first two columns show the likelihood for the respective model, and the third shows the result from the likelihood ratio test, with critical values in parentheses. A value for the test higher than the critical value indicates indicate rejection of the null, restricted model in favour of the alternative, unrestricted model. Although here we show only three specific models (constant A_0 and Λ, constant A_0 and time-varying Λ, time-varying A_0 and constant Λ), similar results hold also for all the models above the dashed line in Figure 2, namely a model with time-varying A_j is always preferred to the corresponding (nested) model with constant A_j.

Among the models that are rejected, one deserves specific mention: the red dot in the lower right corner of Figure 2 indicates the likelihood of a model where A_0 and A_j are assumed constant, and Λ is allowed to change in every subperiod. We deem this particular

Table 2: Log-likelihoods from different model specifications.

Model	A_j constant	A_j time-varying	LR Test
Constant model	-407.82	-298.16	219.32 (72.15)
Time-varying Λ, constant A_0	-294.95	-196.66	196.6 (74.47)
Time-varying A_0, constant Λ	-354.37	-198.08	312.58 (72.15)

Note: The last column shows values for the LR tests, with critical values in parentheses. A value for the test higher than the critical value indicates indicate rejection of the null, restricted model in favour of the alternative, unrestricted model.

model relevant given that its time assumptions are the same as Brunnermeier et al. (2021). We name it "time-varying volatility (TVV) only" model, and note that its likelihood falls way below the threshold line.[8] Figure 4 shows the unconditional variances and correlation for this model, proving further its inability to fit the data dynamics, particularly with regards to the correlation line.

4.2 Permutations

One feature of identification through heteroskedasticity is that shocks are identified only up to sign and column permutation. For a given number of variables (and equations), n, there exist $n!$ permutation matrices (including the identity matrix). Consider the following simplified system of equations, with two variables (y_1 and y_2) and no lags:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = A_0^{-1}\Lambda^{1/2}\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} \tag{9}$$

where

$$A_0^{-1} = \begin{bmatrix} 1 & \alpha \\ \gamma & 1 \end{bmatrix}^{-1} = \frac{1}{1-\alpha\gamma}\begin{bmatrix} 1 & -\alpha \\ -\gamma & 1 \end{bmatrix}$$

and

$$\Lambda^{1/2} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}.$$

Equation (9) is observationally equivalent to:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = A_0^{-1*}\Lambda^{1/2*}\begin{bmatrix} \varepsilon_2 \\ \varepsilon_1 \end{bmatrix} \tag{10}$$

[8] Note that for this model we estimate fewer parameters than for all other models of Figure 2, so while the scale of its likelihood might not be comparable to the other models, we adjust the degrees of freedom when performing the likelihood ratio test, and the TVV only model does not pass the test.

where $A_0^{-1*} = (HPA_0)^{-1}$ and $\Lambda^{1/2*} = HP\Lambda^{1/2}P^{-1}$, P is a permutation matrix, $P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and H is a diagonal rescaling matrix such that the diagonal elements of A_0^* are equal to ones, $H = \begin{bmatrix} \frac{1}{\gamma} & 0 \\ 0 & \frac{1}{\alpha} \end{bmatrix}$. Finally, $HP = \begin{bmatrix} 0 & \frac{1}{\gamma} \\ \frac{1}{\alpha} & 0 \end{bmatrix}$. Note that the order of the shocks in this "permuted model" has swapped.[9]

This is the common issue of permutation within the identification through heteroskedasticity approach. In models in which there is only time variation in Λ, there can be no permutation from one period to the next. Once we also allow time variation in parameters, additional permutations become possible. Specifically, if all coefficients vary, then the order of equations can change between periods.

To pick among all possible permutations, we minimise distance to a "baseline" model. As a baseline model, we choose one where the number of permutations is limited, namely one of the two Choleski models.[10] The triangular structure of this model reduces the possibility to swap equations across periods. We compare each model's impulse response functions to those of the baseline model, and find the permutation which minimises the distance between each model's IRFs and the baseline.[11] As a measure of distance we use the weighted average distance of the IRFs per period and variable, where the weights are the inverse of the standard deviation of each variable in each period. The IRF horizons are given equal weight up to horizon $h = 20$, and zero weight after that. Considering alternative baseline models or measures of distance does not substantially change our conclusions.

4.3 Structural Results

After selecting the subset of models able to replicate the variance-covariance dynamics, and re-ordering the shocks in each model such that they are comparable, we can look at structural results across models. We first investigate impulse responses, enabling us to label the structural shocks from an economic perspective; we then look at individual model elements, namely the shock volatilities, the impact effects, and the lagged effects. These results are based on the maximum likelihood point estimate of each model, and therefore represent results across models, where each model is roughly equally likely, in the sense that it passed the reduced form test of Section 4.1.

[9]See Lanne et al. (2017) for a further explanation of permutations.

[10]Whilst it is irrelevant which of the two Choleski model we pick, we show here results for the lower triangular Choleski decomposition.

[11]This approach is similar to impulse response matching methods used to match impulse responses from DSGE models to those from VARs (e.g. Christiano et al., 2005).

4.3.1 Impulse response functions

Figure 5 shows the cross-model dispersion of impulse responses in each of the four periods, with responses to a one standard deviation shock. The dark grey area includes 68% of all models and the light grey 95% of them. As mentioned before, these are cross-model results and they should not be interpreted as traditional confidence bands: each model is equally likely. The dashed red lines indicate median, minimum, and maximum IRF.

Identification through heteroskedasticity solves the identification problem, but it does not label the shocks it identifies. We now analyze the impulse response functions in order to name the two shocks as a real and a financial shock, respectively. Our labelling is based on the following considerations. First of all, the presence of a substantial response on impact: it seems natural to require that a real shock has a non-zero impact on the real variable, and that a financial shock affects the financial variable on impact.[12] In addition to a relatively small contemporaneous effect on IP, the shock in the second column has more of a delayed effect on IP compared to the first column. This provides additional motivation for labeling this shock the financial shock: it is fairly generally accepted that changes in interest rates reach their maximal effects on real activity with a delay. These dynamics also stand in contrast to the first shock where the effects are both immediate and long lasting, giving additional support for labeling the first shock as the real shock.

These IRF are helpful in labeling shocks, but combine the effects of changing contemporaneous and lagged parameters as well as structural shock volatilities. We now disentangle the role of those more clearly.

4.3.2 Structural coefficients

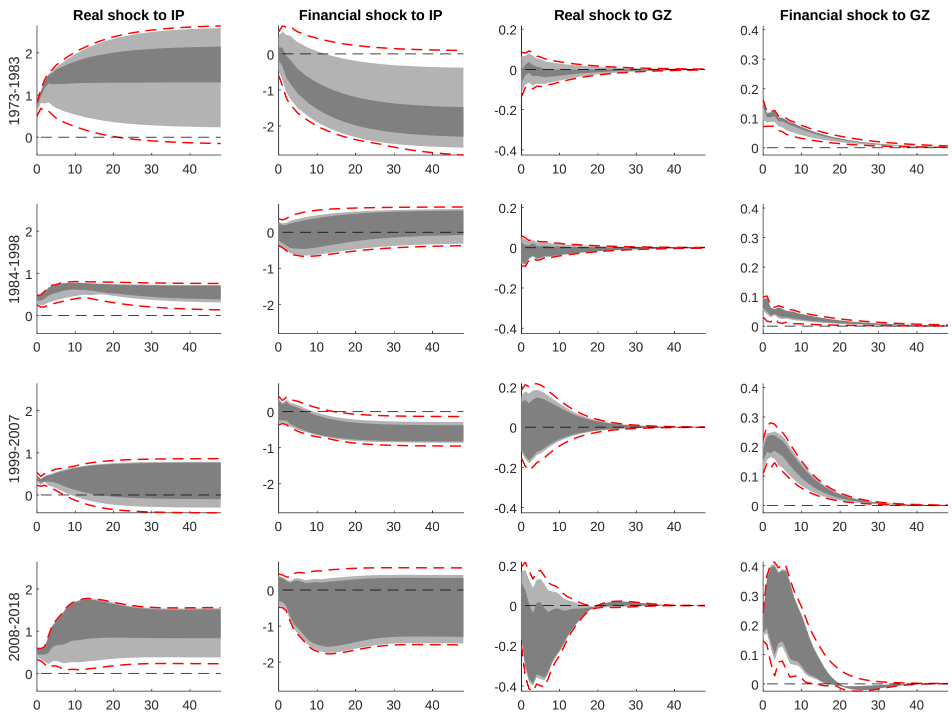
We now look at cross-model structural results, namely structural shock volatilities, impact and lagged effects. Figures 6-9 plot distributions and scatter plots of shock volatilities and transmission mechanisms across all selected models (all models whose likelihood is above the dashed line in Figure 2). In this Section we merely describe the estimated distributions across all models, and their changes over time. In Section 5 we discuss the structural implications of these distributions at length.

Figure 6 shows the cross-model distribution for the volatilities of the real and the financial shock, respectively in the upper and lower panel.[13] Time variation in shock volatilities is

[12]These contemporaneous effects are estimated to be in a pretty tight range across models, which helps in selecting permutations across models.

[13]The distributions for both Figure 6 and 7 are obtained by fitting a Kernel density to the elements of interest across all selected models.

Figure 5: Cross-model dispersion of impulse response functions.



Note: The dark grey area collects 68% of the models, the light grey 95%, and the dashed lines are the minimum and maximum across models. For IP the Figure shows the cumulated response.

clearly visible, confirming the well established result that US macroeconomic and financial shocks exhibited heteroskedasticity since the '70s. More specifically, real shock volatility decreased markedly from period one to two, and only increased back in period four, namely the beginning of the GFC, without reaching pre-Great Moderation levels. Financial shock volatility, on the other hand, was low in the first two periods, and started to increase in the early '00s and to a larger extent in the GFC period.

Figure 7 shows in the two panels cross-model distributions for the contemporaneous effect of a unit financial shock on the real variable and a unit real shock on the financial variable. By investigating the effect of a unit shock, we isolate the role of changes in parameters, thus excluding the role of Λ. By looking at the contemporaneous impulse response, we exclude the role of lagged coefficients A_j, $j > 0$. Therefore this figure zooms in on the off-diagonal elements of A_0. The first result to note is that the curves span both signs in the horizontal axis, pointing at a undetermined sign for the impact effect. Second,

Figure 6: Distributions of shock volatilities for all selected models, per subperiod.

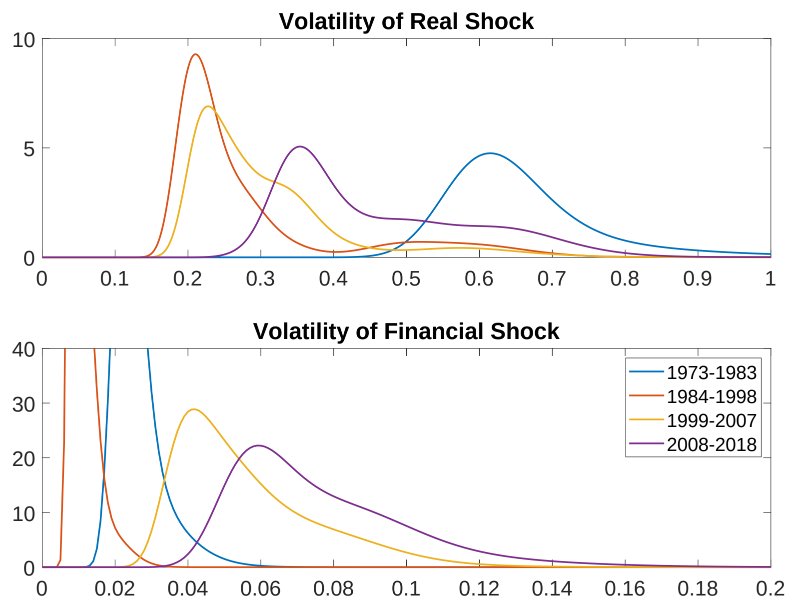


Figure 7: Distributions of structural coefficients for all selected models, per subperiod.

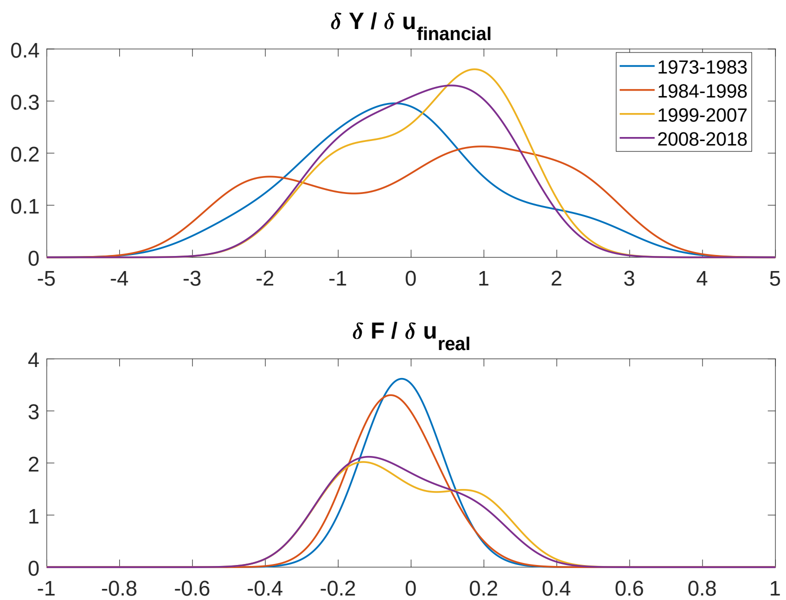


Figure 8: Scatter plot of impulse response functions for all selected models, per subperiod, h=0.

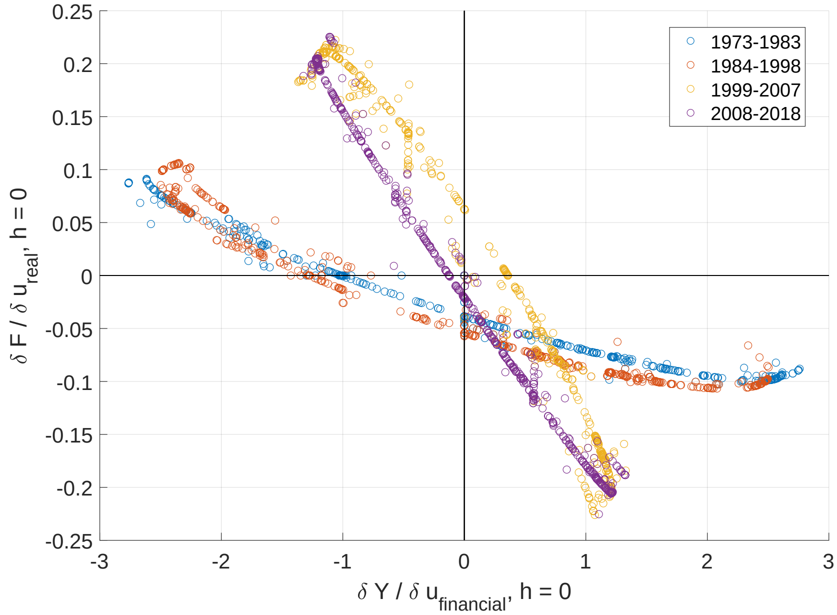
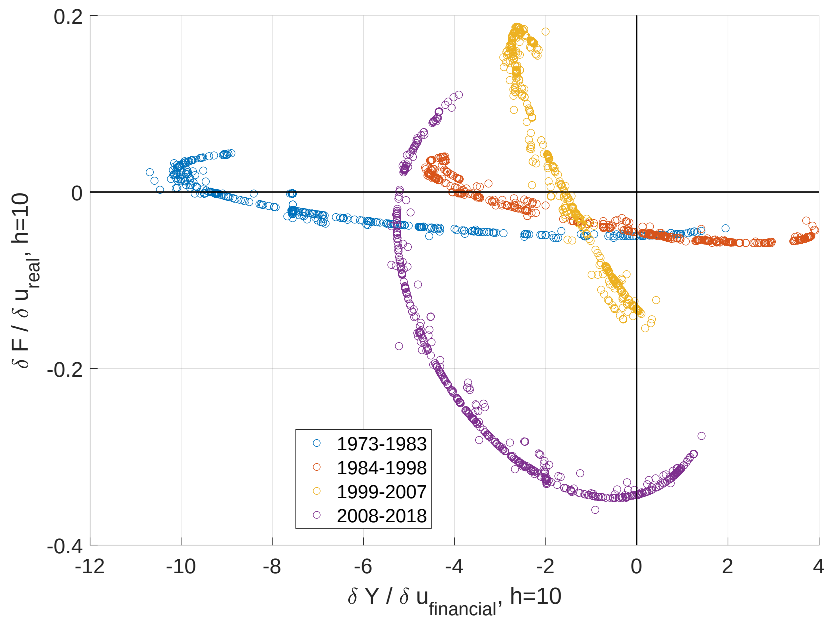


Figure 9: Scatter plot of impulse response functions for all selected models, per subperiod, h=10.



time variation is present and it goes in opposite directions for the two effects: the financial on real impact in the upper panel diminished between period two and three, where the tails of the distributions clearly shrink. The real on financial impact, visible in the lower panel of Figure 7, increased between the second and third period, as can be noticed by the thicker tails of the yellow and purple distributions.

Figure 8 plots the two contemporaneous effects against one another. It illustrates how any given model captures the co-movement between the macro and financial variable. Recall from Figure 1 that the unconditional correlation between the two variables is always negative, but not extremely so: it fluctuates between -.2 and -.5. While that correlation includes lower frequency co-movement, it is instructive to view it through the lens of the two short term feedbacks in Figure 8.

By and large the models populate three quadrants. The upper right quadrant is close to empty, and that is easily understood in light of the negative correlation in Figure 1: models which feature two positive feedbacks imply a positive correlation, which is counterfactual. To match the negative correlation in the data one needs at least one negative elasticity.[14] The two diagonal quadrants have one of the feedbacks negative. These are models in which either the real shock has a positive impact on the financial variable, or the financial shock has a positive impact on the real variable, but not both. These models succeed in capturing the negative unconditional co-movement of Figure 1 by mixing a negative conditional impulse response with a positive one. To a rough approximation, so long as the variance caused by the shock with positive covariance is not too big, these models are able to generate a negative unconditional correlation. The lower left quadrant has both feedbacks negative. In these models, the real variable responds negatively to a financial shock and the financial variable responds negatively to a real shock.

The above illustrates how models with different signs of feedbacks generate the co-movement patterns of Figure 1, depending on which quadrant they locate in. But Figure 8 reveals more. It also suggests that there is a clear change in the slope from period two to three: there appears to be an increase in elasticity of the financial to the real sector and at the same time a decrease in the elasticity of the real to the financial sector. It is useful to understand how that too relates to the patterns of Figure 1. A distinct feature of Figure 1 is that from period two to three financial volatility (green line) goes up without causing an increase in macro volatility (blue line). How do models capture that? One possibility is by increasing the volatility of financial shocks. This immediately generates the jump in the green line. To prevent macro volatility from going up, however, these models must

[14]Throughout the text, we use the term “elasticity” and “feedback” interchangeably.

feature a financial-to-real feedback that is close to zero. If they do not, macro volatility would go up in tandem, which is counterfactual. Yet many models do exhibit a significant financial-to-real feedback. They keep macro volatility constant by reducing the sensitivity of the real variable to the financial variable. The increased financial volatility does not need to originate in increased financial shock volatility. It can also come about as a result of increased real shock volatility at constant real-to-financial feedback. The minor move to the right in the red to yellow distribution in the top panel of Figure 6 suggests this is true for some, but not many models. Another source of the increased financial volatility can be real-to-financial sensitivity (at constant real shock volatility), which is more pervasive: the stark widening of the red to yellow distribution in the bottom panel of Figure 6.[15]

Figure 9, finally, shows the transmission effects for a longer horizon ($h = 10$). In other words, the plot represents the impulse response of the real variable to a financial shock after 10 periods (on the x-axis), and of the financial variable to a real shock after 10 periods (on the y-axis). While so far the discussion was only based on A_0^{-1} and Λ, in this plot the transmission is determined also by the lagged coefficients, A_j. A clear result is that at longer horizons, signs become more negative: the majority of models exhibit negative cross-variable effects for both shocks.

5 Implications for structural models

These results have several interesting implications for structural (DSGE) models. It is worth repeating that despite displaying very different IRF, all the models studied in Section 4 have very similar log-likelihoods; they all succeed in matching the time variation in the data. Further narrowing down the set to a single structural model cannot be done unless one is willing to impose additional restrictions, which is not our objective here. In fact, our discussion below advises against doing so. We here dwell on two specific implications of our structural results, and what they imply for DSGE models of macro-financial interactions. We first argue that the finding that the sign of contemporaneous macro-financial feedbacks is not determined is actually consistent with the wide range of models proposed in the literature. We then discuss what the estimated time variation in macro-financial feedbacks

[15] Understanding how these models capture the time-varying correlation also sheds light on why a model with time variation only in shock volatility underperforms. Since feedbacks are assumed constant, the "TVV only" model has only one way to generate time-varying correlation, which is by changing relative volatilities of shocks through time. It must therefore seek a compromise between matching two volatilities and one correlation, but has only two parameters to do so, in λ_t. Figure 4 shows that compromise implies that the TVV model misses virtually all time variation in the correlation.

suggests for DSGE models and channels.

5.1 Feedback signs

Figures 7 and 8 make clear that the signs of the macro-to-financial and financial-to-macro feedback are not determined.

A positive co-movement between financial spreads and macro variables may seem counter to models of financial frictions. A simple Bernanke, Gertler and Gilchrist (1999) (BGG henceforth) type framework, for example, would suggest that both cross-responses ought to be negative. On the one hand, a positive real shock (e.g. aggregate demand) improves firms' financial conditions and thus reduces the spread. On the other hand, a financial shock (e.g. reduction in firm net worth) increases the spread firms pay for investment and will thus reduce economic activity. In terms of the (short term) scatter plot in Figure 8, DSGE models with these mechanisms in place would locate in the lower left (negative, negative) quadrant. Our estimates suggest that indeed there are successful empirical models with those features.

Yet the bulk of estimated models locate in either the upper left or lower right quadrant. Are these estimated models inconsistent with models of financial frictions? Consider Table 3, which contains a (by no means exhaustive) sample of DSGE models of macro-financial interactions. The table contains information on the sign of (short term) impulse responses of financial spreads to macro shocks (Column 1) and the response of macro variables to financial shocks (Column 2).

The first couple of rows of Column 1 confirm the aforementioned intuition in typical models of the financial accelerator: *macro shocks* induce negative co-movement between spreads and macroeconomic activity, indicated by the negative sign for e.g. BGG, Gertler and Karadi (2011). However, not all shocks and models imply that positive real shocks will reduce spreads. The Carlstrom and Fuerst (1997) model implies positive co-movement. In Cúrdia and Woodford (2010), some of the real shocks induce positive co-movement. De Graeve (2008) illustrates that the cyclicality of spreads can change when the BGG model is extended with Smets-Wouters style shocks and frictions (BGG+SW in Table 3). Faia and Monacelli (2007) show that the cyclicality of the Carlstrom-Fuerst model can change. The Jermann and Quadrini (2012) model implies that spreads reduce following a negative productivity shock. Taken together, Column 1 highlights that theories of financial frictions imply that positive macro shocks can lead to both higher and lower financial spreads. The response depends on the particular type of financial friction, the type of

Table 3: A sample of contemporaneous IRF in structural models

	$\frac{\partial F}{\partial u_{real}}$	$\frac{\partial Y}{\partial u_{fin}}$
Bernanke-Gertler-Gilchrist	<0	<0
Gertler-Karadi	<0	<0
Carlstrom-Fuerst	>0	.
Jermann-Quadrini	>0	<0
Curdia-Woodford	$\gtrless 0$	<0
BGG + SW	$\gtrless 0$	.
Faia-Monacelli	<0	.
Justiniano-Primiceri-Tambalotti	.	>0
Bordalo-Gennaioli-Shleifer	.	$\gtrless 0$

Note: a positive macro shock is one that increases GDP, a positive financial shock is one that increases the financial spread. Different models may have different and multiple real shocks.

macro shock, and real/nominal frictions elsewhere in the economy.

Column 2 reviews the cyclicality of spreads conditional on financial shocks, which received more attention since the GFC. The typical financial accelerator models suggest that financial shocks that lead to higher spreads will cause GDP to fall, which is consistent with conventional wisdom. The Jermann and Quadrini (2012) model, too, leads to a countercyclical financial spread. Yet the literature has also proposed alternative theories for financial crises, which one might summarily characterise as boom-bust views. According to these, in the boom phase there is scope for investment (or GDP) and financial spreads to rise simultaneously. There are different potential sources of that positive co-movement. One class of models suggest that the extensive margin of borrowers, or risk, is important. Credit demand or supply are high in the boom phase, leading to more yet riskier investment. The table contains two such examples. The first is Justiniano, Primiceri and Tambalotti (2010), in which excess credit supply induces banks to lend more. While credit may become cheaper for a given borrower (i.e. the intensive margin), increased credit supply also implies access to credit for less creditworthy borrowers (i.e. the extensive margin). As riskier borrowers enter the pool, financial spreads rise while investment increases. A second class of models suggests behavioral sources for high credit demand: borrowers have wrong (e.g. overly optimistic) expectations. Consumers or firms are willing to borrow at higher spreads because they are overly optimistic about future (e.g. house price) realizations. An example of this class of models is the work of Bordalo, Gennaioli and Shleifer (2018). Note that these theories predict that the positive co-movement in the boom is followed by negative co-movement in the bust-phase. This too, is consistent with our results: Figure 9 shows that at longer horizons the majority of models are located in the (negative, negative) quadrant. Together, Table 3 suggests that structural models in the DSGE literature all fall within the realm of estimated macro-financial interactions. On the basis of the data studied here, all these models have the scope to generate the feedbacks we estimate. This is consistent with the wide variety of DSGE models that have been used to understand the GFC, and the claim they match important features of it.

5.2 Time variation

The results of Section 4 reveal that only models that feature time variation in *both* parameters *and* volatilities can replicate the time-varying (co-)variance patterns in the data. It is noteworthy that most DSGE models of macro-financial interactions feature *either* stochastic volatility (e.g. Fernández-Villaverde et al., 2015) *or* time variation in parameters (e.g. Guerrieri and Iacoviello, 2017), but not often both. Our results regarding structural stochastic volatility are generally in agreement with empirical models for both real and financial shocks (e.g. Brunnermeier et al., 2021). While pinpointing an exact structural model cannot be done on the basis of our results, they do suggest that there is particular promise for models in which time variation in the structure of the economy is present. Specifically, the evidence suggests that the financial impact of real shocks became stronger in the second part of our sample, while the real impact of financial shocks became smaller. While this may seem odd at first glance given the immense real impact of the GFC, that came about in the presence of an increased volatility of financial shocks. As such, a narrative in which the economy became more resilient as a result of financial innovation/deregulation in the '80s and '90s, but thereby also became more vulnerable to (perhaps new types of) financial shocks seems plausible. The increased elasticity of financial variables to macro aligns well with models arguing that financial constraints became more relevant. This narrative is consistent with the changes in the shape of the cross-model distributions of macro-financial feedbacks and volatilities over time. This is not to say that alternative narratives and models are ruled out by our results: so long as a model's feedbacks and shock volatilities over time move within the distributions shown, our results suggest it may well be able to capture macro-financial covariances over time.

We are hesitant to draw more specific structural conclusions from our results on time variation, because there are multiple conceptual interpretations possible. On the one hand, one could look at the time variation in a single estimated model, and interpret the change in parameters as a change in the sensitivity for a given structural mechanism. On the other hand, it is equally possible to think of time variation within a single estimated model as reflecting changing importance of different structural mechanisms, e.g. models that switch between frictions over time.

6 Extension: a three-variable VAR

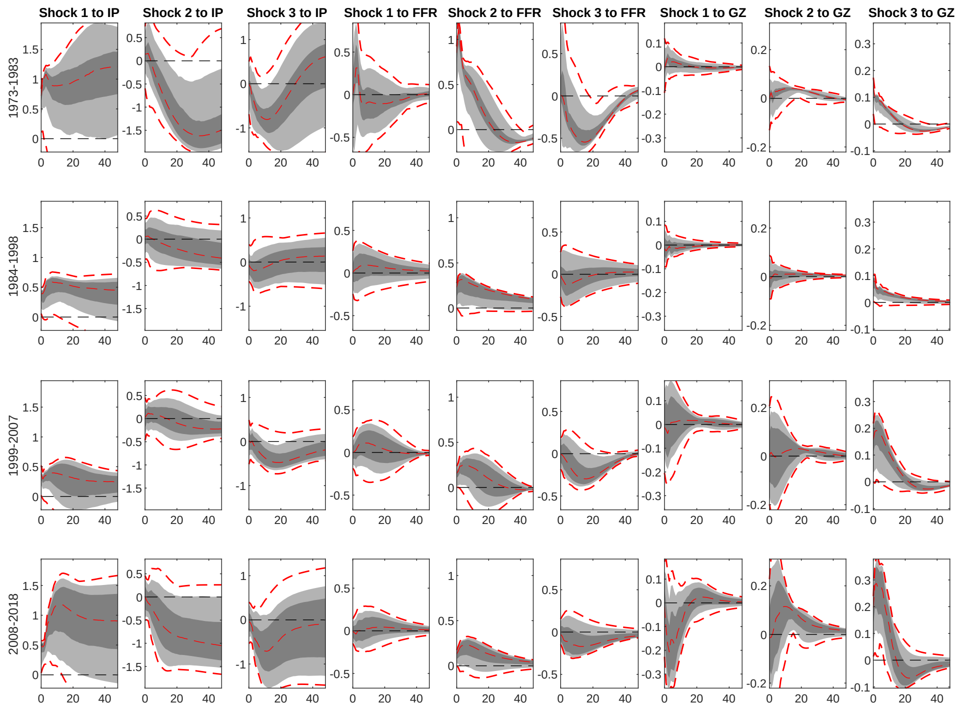
A caveat to our analysis is the fact that our specification is a small system, with only two variables. While a number of recent studies also focus on two-variable systems (e.g. Adrian et al., 2019, 2021), it is well known that missing relevant variables or shocks can generate undesirable effects. Particularly, missing out on a relevant variable may be absorbed by parameter and volatility estimates changing over time in a smaller model, even if there was no time variation in the data generating process. While this is always a relevant concern for any empirical model, we feel the results from our two variable specification generate substantial insight in their own right. The volatility and covariance patterns of Figure 1 are a feature of the data (see also Appendix A) and need to be matched also by bigger models. The coherence between different sources of time variation is much more difficult to evaluate in larger models, yet the same effects we illustrate can be at play.

In this Section we present results for a specification with three variables, and show that our results do not change substantially. One point to note is that increasing the number of variables (and therefore of coefficients) implies a much larger set of possible models (refer to subsection 2.3.1 for an explanation of how we calculate all possible combinations). We sideline this issue by estimating a random sample of possible models, with the assumption that results can be generalised for the whole "population" of models. As a third variable, we add the federal funds rate, complementing it with the shadow rate by Wu and Xia (2016) for the zero lower bound period.

Figures 10, 11 and 12 present cross-model structural results for the three-variable specification. Results for the real and the financial shocks are broadly similar to the baseline case. In addition, we label the additional shock (which we order second) "monetary policy shock", after inspecting the impulse response functions (Figure 10): the second shock has a delayed negative effect on industrial production, an immediate positive effect on the federal funds rate, and a delayed positive effect on the GZ spread.

The model distributions in Figure 11 are broadly consistent with the two variable case: higher financial shock volatilities since the early 2000s, low real shock volatility in the Great Moderation period, coupled with larger effects of real shocks on the financial sector at the

Figure 10: Cross-model dispersion of impulse response functions, 3-variable specification.



Note: The dark grey area collects 68% of the models, the light grey 95%, and the dashed lines are the minimum and maximum across models.

beginning of the century; additionally, the volatilities of the monetary policy shock behave as expected given the Great Moderation and zero lower bound periods, with large volatility only in the first subperiod (Great Inflation). Finally, Figure 12 qualitatively confirms the baseline findings: the same quadrants are populated. Because the system is bigger, the close connection between the correlation in Figure 1 and contemporaneous impulse responses is less tight than described in Section 4.3.2 (since IRF of other variables to other shocks also affect the unconditional correlation). This causes the models to locate in clouds more than the almost linear locations in Figure 8. Yet the qualitative conclusions remain.

Figure 11: Distributions of structural coefficients and of shock volatilities for selected 3-variable models, per subperiod.

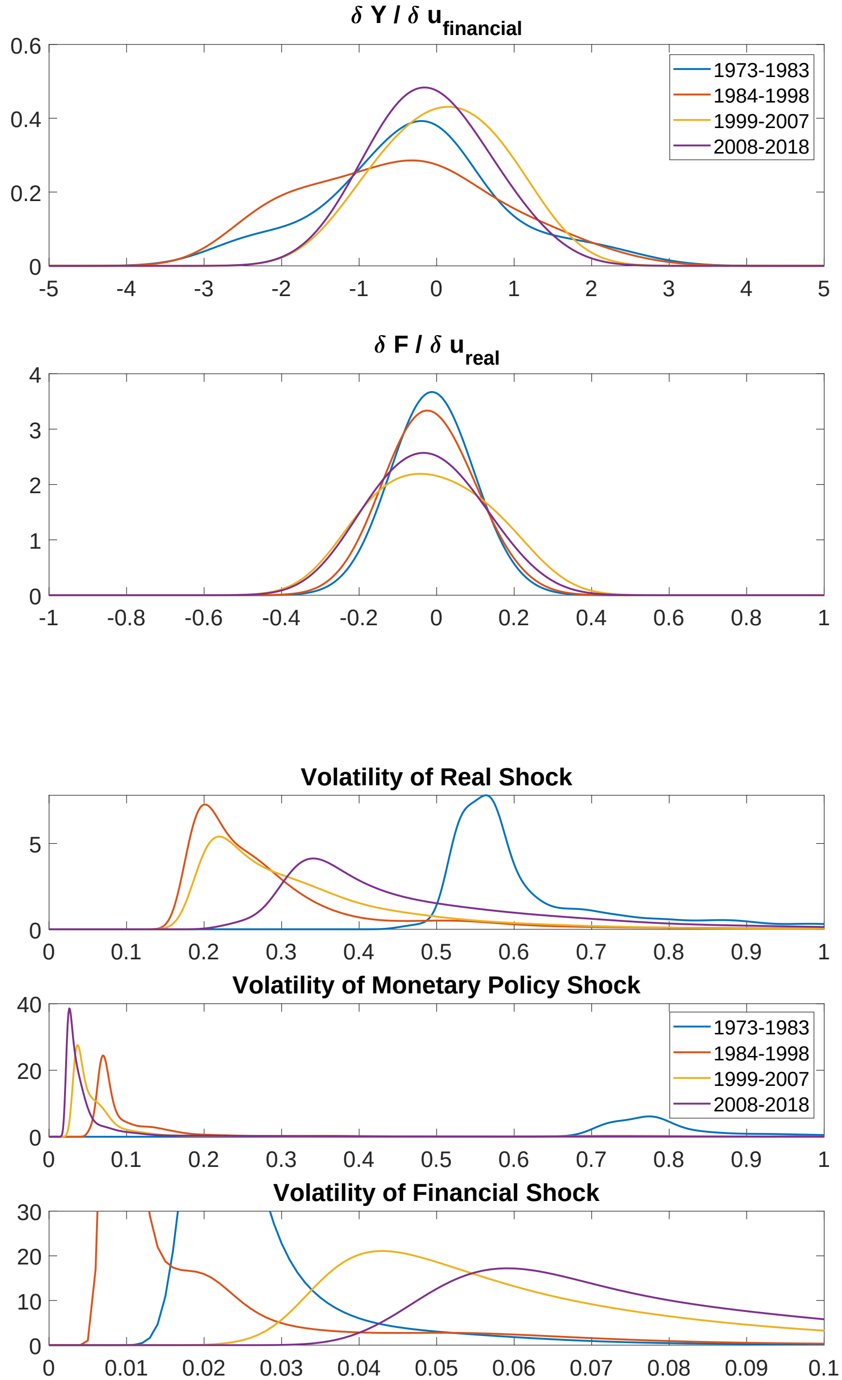
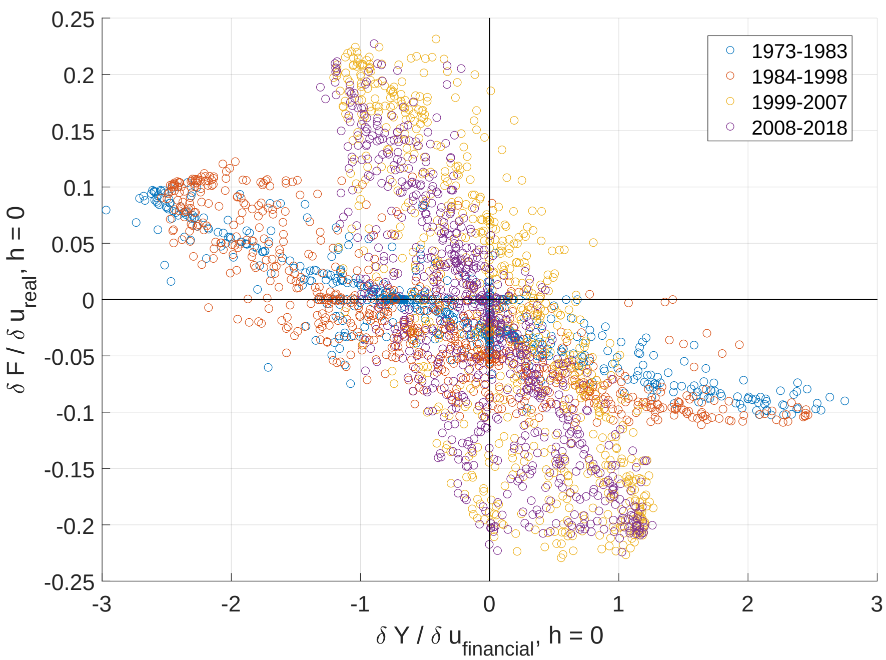
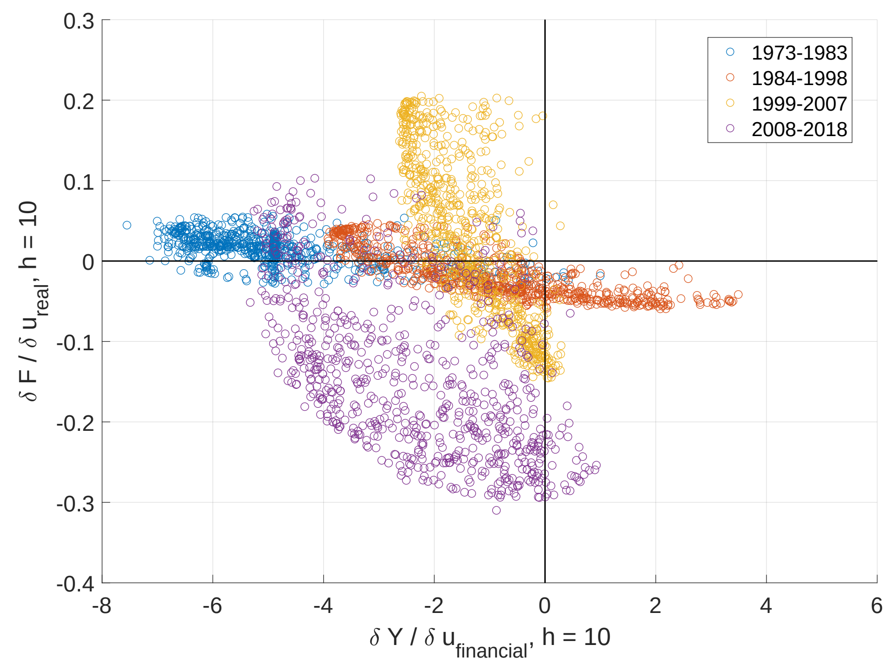


Figure 12: Scatter plot of impulse response functions for selected 3-variable models, per subperiod, h=0 and h=10.





7 Further discussion and robustness

7.1 Alternative identification approaches

The results also shed light on what alternative approaches to identification imply. They will typically boil down to picking a point or a quadrant in Figure 8 a priori. The fact that the plausible models span across multiple quadrants implies the data as such cannot distinguish between very different theoretical models. Table 3 suggests that the spectrum of theories on financial frictions does not obviously pin down any particular quadrant (let alone a single point) a priori. This suggests that alternative approaches such as point restrictions (e.g. Choleski) or sign restrictions will not credibly identify shocks in this setting. All they would do is constrain regions that are plausible both a priori (cf. Table 3) and a posteriori (Figure 2). It is exactly in light of difficulties with traditional approaches to identification that identification through heteroskedasticity was proposed by Rigobon (2003). What our approach and results show is the power of that approach can be extended to models in which not just variances, but also parameters are changing over time.

7.2 Labelling and permutation algorithm

As described in Section 2, identification through heteroskedasticity (and our extension of it) does not pin down the interpretation of shocks. The specific algorithm we use to choose among various permutations, the labelling of the shocks and the choice of baseline model can be contested. Yet there are a number of reasons that suggest alternative approaches to permutations will not overturn our results.

First, the exact choice of baseline model does not have a big impact on our results. Second, we have thoroughly investigated alternative approaches to permutations (e.g. alternative weights and horizons in impulse response matching) and find no substantive changes in conclusions. Third, other than normalization, alternative permutations or choice of labelling essentially swap real and financial shocks with one another. In terms of Figure 8, an alternative label (i.e. calling the real shock financial and vice versa) effectively boils down to flipping models around the 45-degree line. That means that alternative shock-labels or permutations will never permute all models into a single quadrant. While one might hope to reduce the two diagonal quadrants into one, we have considered such permutations and it is very obvious from the IRF across these permuted models that they mix up two distinct types of shocks.

Note also that the permutation problem can actually become clearer in bigger systems, as adding variables and shocks can help pin down certain shocks and equations more clearly, see Section 6.

8 Conclusions

We generalize "identification through heteroskedasticity" to a setting in which both parameters and shock volatilities change over time. Both types of time variation are required to match macro-financial (co-)variances in the data. Our generalization leads to set rather than point identification. The set of empirically successful identified models is informative for understanding macro-financial feedbacks. It reveals why different theories of financial frictions, with very different feedbacks, can all succeed in explaining financial crises. Traditional approaches to identification needlessly exclude some of these theories. Our estimates suggest particular promise for DSGE models which feature an increase in the sensitivity of financial spreads to macro variables, while macro variables themselves became less sensitive to financial spreads. Just like macro-financial feedbacks, structural shock volatility is also time-varying.

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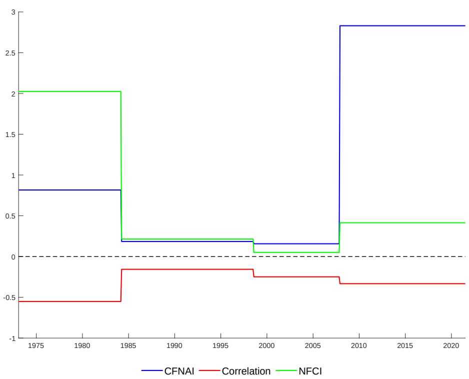
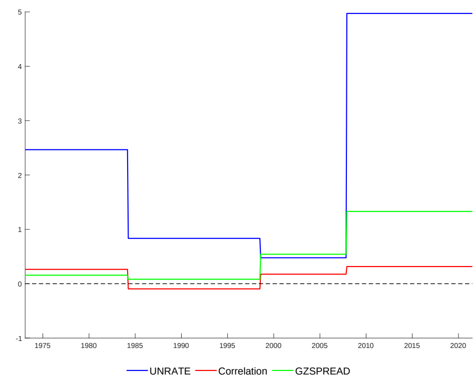
A Alternative indicators

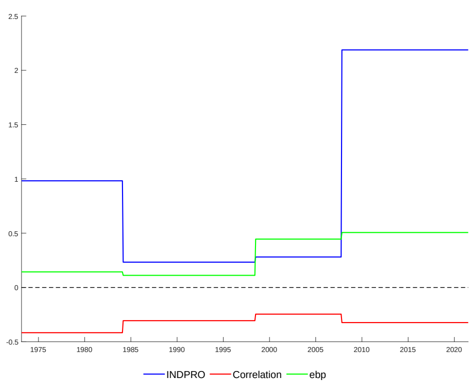
Figure 13: Unconditional variances and correlation of various indicators



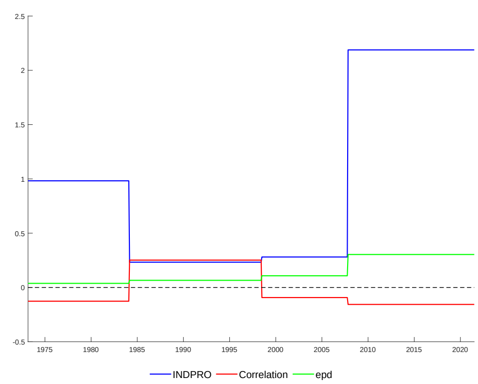
(a) Chicago Fed National Activity Index (CFNAI) and National Financial Conditions Index (NFCI)



(b) Unemployment rate and GZ spread

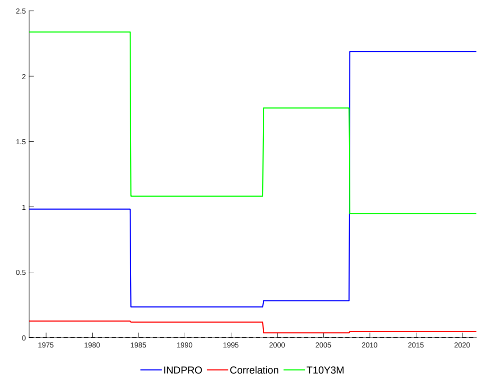


(c) Industrial production and Excess Bond Premium (Gilchrist and Zakrajšek, 2012)

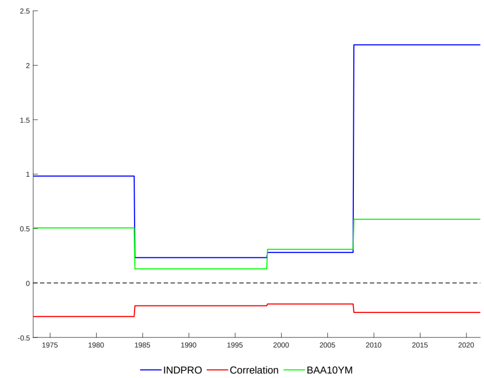


(d) Industrial production and Expected Probability of Default (Gilchrist and Zakrajšek, 2012)

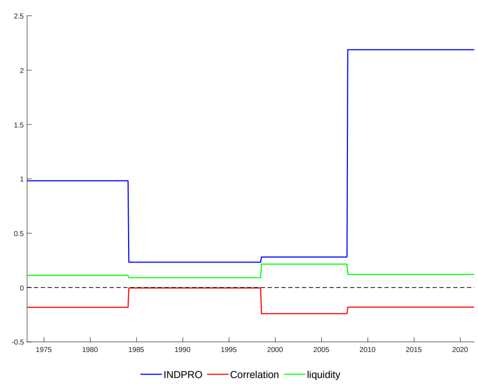
Figure 13: Unconditional variances and correlation of various indicators - continued



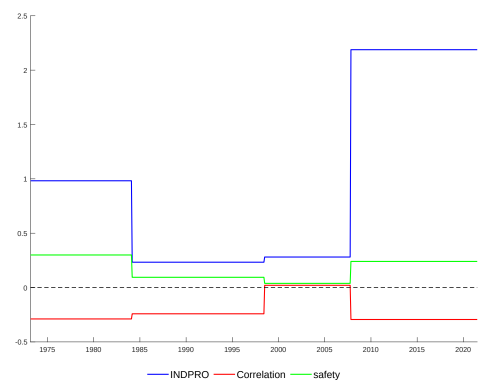
(e) Industrial production and Term Spread



(f) Industrial production and BAA Spread



(g) Industrial production and Liquidity Spread



(h) Industrial production and Safety Spread